CHAPTER 1

REAL NUMBERS

SUMMARY

1. **Algorithm :** An algorithm means a series of well defined step which gives a procedure for solving a type of problem.
2. **Lemma :** A lemma is a proven statement used for proving another statement.
3. **Fundamental Theorem of Arithmetic :** Every composite number can be expressed (factorised) as a product of primes and this factorisation is unique apart from the order in which the prime factors occur.
4. If p is prime number and p divides a^2, then p divides a, where a is a positive integer.
5. If x be any rational number whose decimal expansion terminates, then we can express x in the form $\frac{p}{q}$, where p and q are co-prime and the prime factorisation of q is of the form $2^n \times 5^m$, where n and m are non-negative integers.
6. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form $2^n \times 5^m$, where n and m are non-negative integers, then x has a decimal expansion which terminates.
7. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form $2^n \times 5^m$, where n and m are non-negative integers, then x has a decimal expansion which is non-terminating repeating (recurring).
8. For any two positive integers p and q, HCF $(p, q) \times$ LCM $(p, q) = p \times q$.
9. For any three positive integers p, q and r,

$$\text{LCM } (p, q, r) = \frac{p \times q \times r \times \text{HCF}(p, q, r)}{\text{HCF}(p, q) \times \text{HCF}(q, r) \times \text{HCF}(p, r)}$$

$$\text{HCF } (p, q, r) = \frac{p \times q \times r \times \text{LCM}(p, q, r)}{\text{LCM}(p, q) \times \text{LCM}(q, r) \times \text{LCM}(p, r)}$$

ONE MARK QUESTIONS

Multiple Choice Questions

1. The sum of exponents of prime factors in the prime-factorisation of 196 is

(a) 3 (b) 4

(c) 5 (d) 2

Ans : [Board 2020 OD Standard]

Prime factors of 196,

$$196 = 4 \times 49$$
$$= 2^2 \times 7^2$$

The sum of exponents of prime factor is $2 + 2 = 4$.

Thus (b) is correct option.

2. The total number of factors of prime number is

(a) 1 (b) 0

(c) 2 (d) 3

Ans : [Board 2020 Delhi Standard]

There are only two factors (1 and number itself) of any prime number.

Thus (c) is correct option.

3. The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140 (b) 12, 420

(c) 3, 420 (d) 420, 3

Ans : [Board 2020 Delhi Standard]

We have
$$12 = 2 \times 2 \times 3$$
$$21 = 3 \times 7$$
$$15 = 3 \times 5$$
$$\text{HCF}(12, 21, 15) = 3$$
$$\text{LCM } (12, 21, 15) = 2 \times 2 \times 3 \times 5 \times 7 = 420$$

Thus (c) is correct option.

4. The decimal representation of $\frac{11}{2^3 \times 5}$ will

(a) terminate after 1 decimal place

(b) terminate after 2 decimal place

(c) terminate after 3 decimal places

(d) not terminate

Ans : [Board 2020 SQP Standard]

We have $$\frac{11}{2^3 \times 5} = \frac{11}{2^3 \times 5^1}$$

Denominator of $\frac{11}{2^3 \times 5}$ is of the form $2^m \times 5^n$, where m, n are non- negative integers. Hence, $\frac{11}{2^3 \times 5}$ has terminating decimal expansion.

Now $$\frac{11}{2^3 \times 5} = \frac{11}{2^3 \times 5} \times \frac{5^2}{5^2}$$

$$= \frac{11 \times 5^2}{2^3 \times 5^3} = \frac{11 \times 25}{10^3} = 0.275$$

So , it will terminate after 3 decimal places.

Thus (c) is correct option.

5. The LCM of smallest two digit composite number and smallest composite number is

(a) 12 (b) 4

(c) 20 (d) 44

Ans : [Board 2020 SQP Standard]

Smallest two digit composite number is 10 and smallest composite number is 4.

$$\text{LCM }(10, 4) = 20$$

Thus (c) is correct option.

6. HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, then the other number is

(a) 36 (b) 35

(c) 9 (d) 81

Ans : [Board 2020 OD Basic]

Let y be the second number.

Since, product of two numbers is equal to product of LCM and HCM,

$$54 \times y = \text{LCM} \times \text{HCF}$$

$$54 \times y = 162 \times 27$$

$$y = \frac{162 \times 27}{54} = 81$$

Thus (b) is correct option.

7. HCF of 144 and 198 is

(a) 9 (b) 18

(c) 6 (d) 12

Ans : [Board 2020 Delhi Basic]

Using prime factorization method,

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$= 2^4 \times 3^2$$

and $$198 = 2 \times 3 \times 3 \times 11$$

$$= 2 \times 3^2 \times 11$$

$$\text{HCF}(144, 198) = 2 \times 3^2 = 2 \times 9 = 18$$

Thus (b) is correct option.

8. 225 can be expressed as

(a) 5×3^2 (b) $5^2 \times 3$

(c) $5^2 \times 3^2$ (d) $5^3 \times 3$

Ans : [Board 2020 Delhi Basic]

By prime factorization of 225, we have

$$225 = 3 \times 3 \times 5 \times 5$$

$$= 3^2 \times 5^2 \text{ or } 5^2 \times 3^2$$

Thus (c) is correct option.

9. The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal?

(a) 2 (b) 4

(c) 5 (d) 1

Ans : [Board 2020 OD Basic]

$$\frac{23}{2^5 \times 5^2} = \frac{23 \times 5^3}{2^5 \times 5^2 \times 5^3}$$

$$= \frac{23 \times 125}{2^5 \times 5^5} = \frac{2875}{(10)^5}$$

$$= \frac{2875}{100000} = 0.02875$$

Hence, $\frac{23}{2^5 \times 5^2}$ will terminate after 5 five decimal places.

Thus (c) is correct option.

10. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after

(a) one decimal place (b) two decimal places

(c) three decimal places (d) four decimal places

Ans : [Board 2020 Delhi Standard]

Rational number,

$$\frac{14587}{1250} = \frac{14587}{2^1 \times 5^4} = \frac{14587}{2^1 \times 5^4} \times \frac{2^3}{2^3}$$

$$= \frac{14587 \times 8}{2^4 \times 5^4} = \frac{116696}{(10)^4}$$

$$= 11.6696$$

Hence, given rational number will terminate after four decimal places.

Thus (d) is correct option.

11. $2.\overline{35}$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a natural number

Ans : [Board 2020 Delhi Basic]

$2.\overline{35}$ is a rational number because it is a non terminating repeating decimal.

Thus (b) is correct option.

12. $2\sqrt{3}$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a whole number

Ans : [Board 2020 OD Basic]

Let us assume that $2\sqrt{3}$ is a rational number.

Now $2\sqrt{3} = r$ where r is rational number

or $\sqrt{3} = \frac{r}{2}$

Now, we know that $\sqrt{3}$ is an irrational number, So, $\frac{r}{2}$ has to be irrational to make the equation true. This is a contradiction to our assumption. Thus, our assumption is wrong and $2\sqrt{3}$ is an irrational number.

Thus (c) is correct option.

13. The product of a non-zero rational and an irrational number is

(a) always irrational (b) always rational

(c) rational or irrational (d) one

Ans :

Product of a non-zero rational and an irrational number is always irrational i.e., $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ which is irrational.

Thus (a) is correct option.

14. For some integer m, every even integer is of the form

(a) m (b) $m+1$

(c) $2m$ (d) $2m+1$

Ans :

We know that even integers are 2, 4, 6, ...

So, it can be written in the form of $2m$ where m is a integer.

$$m = ..., -1, 0, 1, 2, 3, ...$$

$$2m = ..., -2, 0, 2, 4, 6, ...$$

Thus (c) is correct option.

15. For some integer q, every odd integer is of the form

(a) q (b) $q+1$

(c) $2q$ (d) $2q+1$

Ans :

We know that odd integers are 1, 3, 5, ...

So, it can be written in the form of $2q+1$ where q is integer.

$$q = ..., -2, -1, 0, 1, 2, 3, ...$$

$$2q+1 = ..., -3, -1, 1, 3, 5, 7, ...$$

Thus (d) is correct option.

16. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) is

(a) xy (b) xy^2

(c) x^3y^3 (d) x^2y^2

Ans :

We have $a = x^3y^2 = x \times x \times x \times y \times y$

$$b = xy^3 = x \times y \times y \times y$$

$$\text{HCF}(a, b) = \text{HCF}(x^3y^3, xy^3)$$

$$= x \times y \times y = xy^2$$

HCF is the product of the smallest power of each common prime factor involved in the numbers.

Thus (b) is correct option.

17. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; where a, b being prime numbers, then LCM (p, q) is equal to

(a) ab (b) a^2b^2

(c) a^3b^2 (d) a^3b^3

Ans :

We have $p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

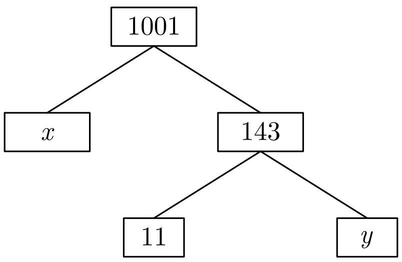
$$\text{LCM}(p, q) = \text{LCM}(ab^2, a^3b)$$

$$= a \times b \times b \times a \times a = a^3 b^2$$

LCM is the product of the greatest power of each prime factor involved in the numbers.

Thus (c) is correct option.

18. The values of x and y in the given figure are



(a) 7, 13 (b) 13, 7

(c) 9, 12 (d) 12, 9

Ans :

$$1001 = x \times 143 \Rightarrow x = 7$$

$$143 = y \times 11 \Rightarrow y = 13$$

Hence $x = 7,\ y = 13$

Thus (a) is correct option.

19. The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

(a) 10 (b) 100

(c) 504 (d) 2520

Ans :

Factor of 1 to 10 numbers

$$1 = 1$$
$$2 = 1 \times 2$$
$$3 = 1 \times 3$$
$$4 = 1 \times 2 \times 2$$
$$5 = 1 \times 5$$
$$6 = 1 \times 2 \times 3$$
$$7 = 1 \times 7$$
$$8 = 1 \times 2 \times 2 \times 2$$
$$9 = 1 \times 3 \times 3$$
$$10 = 1 \times 2 \times 5$$

$$\text{LCM}(1 \text{ to } 10) = \text{LCM}(1,2,3,4,5,6,7,8,9,10)$$
$$= 1 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$$
$$= 2520$$

Thus (d) is correct option.

20. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

(a) an even number (b) an odd number

(c) an odd prime number (d) a prime number

Ans :

$p_1^2 - p_2^2$ is an even number.

Let us take $p_1 = 5$

and $p_2 = 3$

Then, $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

Thus (a) is correct option.

21. The rational form of $0.2\overline{54}$ is in the form of $\frac{p}{q}$ then $(p+q)$ is

(a) 14 (b) 55

(c) 69 (d) 79

Ans :

Let, $x = 0.2\overline{54}$, then

$$x = 0.2545454 \text{ ....} \qquad \text{...(1)}$$

Multiplying equation (1) by 100, we get

$$100x = 25.4545. \text{ ....} \qquad \text{...(2)}$$

Subtracting equation (1) from equation. (2), we get

$$99x = 25.2 \Rightarrow x = \frac{252}{990} = \frac{14}{55}$$

Comparing with $\frac{p}{q}$, we get

$p = 14$

and $q = 55$

Hence, $p + q = 14 + 55 = 69$

Alternative :

$$0.2\overline{54} = \frac{254 - 2}{990} = \frac{252}{990} = \frac{14}{55}$$

Thus (c) is correct option.

22. The rational number of the form $\frac{p}{q}$, $q \neq 0$, p and q are positive integers, which represents $0.1\overline{34}$ i.e., (0.1343434 ....) is

(a) $\frac{134}{999}$ (b) $\frac{134}{990}$

(c) $\frac{133}{999}$ (d) $\frac{133}{990}$

Ans :

$$0.1\overline{34} = \frac{134-1}{990} = \frac{133}{990}$$

Thus (d) is correct option.

23. Which of the following will have a terminating decimal expansion?

(a) $\frac{77}{210}$ (b) $\frac{23}{30}$

(c) $\frac{125}{441}$ (d) $\frac{23}{8}$

Ans :

For terminating decimal expansion, denominator must the form of $2^m \times 5^n$ where n, m are non-negative integers.

Here, $\frac{23}{8} = \frac{23}{2^3}$

Here only 2 is factor of denominator so terminating.

Thus (d) is correct option.

24. If $x = 0.\overline{7}$, then $2x$ is

(a) $1.\overline{4}$ (b) $1.\overline{5}$

(c) $1.\overline{54}$ (d) $1.\overline{45}$

Ans :

We have $x = 0.\overline{7}$

$$10x = 7.\overline{7}$$

Subtracting, $9x = 7$

$$x = \frac{7}{9}$$

$$2x = \frac{14}{9} = 1.555 \text{ ....}$$

$$= 1.\overline{5}$$

25. Which of the following rational number have non-terminating repeating decimal expansion?

(a) $\frac{31}{3125}$ (b) $\frac{71}{512}$

(c) $\frac{23}{200}$ (d) None of these

Ans :

$$3125 = 5^5 = 5^5 \times 2^0$$

$$512 = 2^9 = 2^9 \times 5^0$$

$$200 = 2^3 \times 5^2$$

Thus 3125, 512 and 200 has factorization of the form $2^m \times 5^n$ (where m and n are whole numbers). So given fractions has terminating decimal expansion.

Thus (d) is correct option.

26. The number $3^{13} - 3^{10}$ is divisible by

(a) 2 and 3 (b) 3 and 10

(c) 2, 3 and 10 (d) 2, 3 and 13

Ans :

$$3^{13} - 3^{10} = 3^{10}(3^3 - 1) = 3^{10}(26)$$

$$= 2 \times 13 \times 3^{10}$$

Hence, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.

Thus (d) is correct option.

27. 1. The L.C.M. of x and 18 is 36.

2. The H.C.F. of x and 18 is 2.

What is the number x?

(a) 1 (b) 2

(c) 3 (d) 4

Ans :

LCM $\times$ HCF = First number $\times$ second number

Hence, required number $= \frac{36 \times 2}{18} = 4$

Thus (d) is correct option.

28. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and $\text{LCM}(a, b, c) = 2^3 \times 3^2 \times 5$, then n is

(a) 1 (b) 2

(c) 3 (d) 4

Ans :

Value of n must be 2.

Thus (b) is correct option.

29. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

(a) 240 (b) 1600

(c) 2400 (d) 3600

Ans :

The LCM of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect square number. 1600 is not multiple of 240.

Thus (d) is correct option.

30. $n^2 - 1$ is divisible by 8, if n is

(a) an integer
(b) a natural number
(c) an odd integer
(d) an even integer

Ans :

Let, $a = n^2 - 1$

For $n^2 - 1$ to be divisible by 8 (even number), $n^2 - 1$ should be even. It means n^2 should be odd i.e. n should be odd.

If n is odd, $n = 2k + 1$ where k is an integer

$$a = (2k+1)^2 - 1$$
$$= 4k^2 + 4k + 1 - 1$$
$$= 4k^2 + 4k$$
$$a = 4k(k+1)$$

At $k = -1$, $a = 4(-1)(-1+1) = 0$

which is divisible by 8.

At $k = 0$, $a = 4(0) + (0+1) = 0$

which is divisible by 8.

Hence, we can conclude from above two cases, if n is odd, then $n^2 - 1$ is divisible by 8.

Thus (c) is correct option.

31. When 2^{256} is divided by 17 the remainder would be

(a) 1
(b) 16
(c) 14
(d) None of these

Ans : (a) 1

When 2^{256} is divided by 17 then,

$$\frac{2^{256}}{2^4 + 1} = \frac{(2^4)^{64}}{(2^4 + 1)}$$

By remainder theorem when $f(x)$ is divided by $x + a$ the remainder is $f(-a)$.

Here, $f(x) = (2^4)^{64}$ and $x = 2^4$ and $a = 1$

Hence, remainder $f(-1) = (-1)^{64} = 1$

Thus (a) is correct option.

32. Assertion : $\frac{13}{3125}$ is a terminating decimal fraction.

Reason : If $q = 2^m 5^n$ where m, n are non-negative integers, then $\frac{p}{q}$ is a terminating decimal fraction.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $3125 = 5^5 = 5^5 \times 2^0$

Since the factors of the denominator 3125 is of the form $2^0 \times 5^5$, $\frac{13}{3125}$ is a terminating decimal

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

Thus (a) is correct option.

33. Assertion : 34.12345 is a terminating decimal fraction.

Reason : Denominator of 34.12345, when expressed in the form $\frac{p}{q}$, $q \neq 0$, is of the form $2^m \times 5^n$, where m and n are non-negative integers.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Ans :

$$34.12345 = \frac{3412345}{100000} = \frac{682469}{20000} = \frac{682469}{2^5 \times 5^4}$$

Its denominator is of the form $2^m \times 5^n$, where $m = 5$ and $n = 4$ which are non-negative integers.

Thus both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

34. Assertion : The HCF of two numbers is 5 and their product is 150, then their LCM is 30

Reason : For any two positive integers a and b, HCF (a, b) + LCM $(a, b) = a \times b$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Ans : (c) Assertion (A) is true but reason (R) is false.

We have,

$$\text{LCM}(a,b) \times \text{HCF}(a,b) = a \times b$$

$$\text{LCM} \times 5 = 150$$

$$\text{LCM} = \frac{150}{5} = 30$$

Thus (c) is correct option.

Fill in the Blank Questions

35. If every positive even integer is of the form $2q$, then every positive odd integer is of the form .... where q is some integer.

Ans :

$2q+1$

36. The exponent of 2 in the prime factorisation of 144, is ....

Ans :

4

37. $\sqrt{2}, \sqrt{3}, \sqrt{7}$, etc. are .... numbers.

Ans :

Irrational

38. Every point on the number line corresponds to a .... number.

Ans :

Real

39. The product of three numbers is .... to the product of their HCF and LCM.

Ans :

Not equal

40. If p is a prime number and it divides a^2 then it also divides ...., where a is a positive integer.

Ans :

a

41. Every real number is either a .... number or an .... number.

Ans :

Rational, irrational

42. Numbers having non-terminating, non-repeating decimal expansion are known as ....

Ans :

Irrational numbers

Very Short Answer Questions

43. What is the HCF of smallest primer number and the smallest composite number?

Ans : [Board 2018]

Smallest prime number is 2 and smallest composite number is 4. HCF of 2 and 4 is 2.

44. Write one rational and one irrational number lying between 0.25 and 0.32.

Ans : [Board 2020 SQP Standard]

Given numbers are 0.25 and 0.32.

Clearly $\quad 0.30 = \frac{30}{100} = \frac{3}{10}$

Thus 0.30 is a rational number lying between 0.25 and 0.32. Also 0.280280028000.....has non-terminating non-repeating decimal expansion. It is an irrational number lying between 0.25 and 0.32.

45. If HCF(336, 54) $= 6$, find LCM(336, 54).

Ans : [Board 2019 OD]

$$\text{HCF} \times \text{LCM} = \text{Product of number}$$

$$6 \times \text{LCM} = 336 \times 54$$

$$\text{LCM} = \frac{336 \times 54}{6}$$

$$= 56 \times 54 = 3024$$

Thus LCM of 336 and 54 is 3024.

46. Explain why 13233343563715 is a composite number?

Ans : [Board Term-1 2016]

The number 13233343563715 ends in 5. Hence it is a multiple of 5. Therefore it is a composite number.

47. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of $(a+b)$.

Ans : [Board Term-1 2014]

Here a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. The least prime factor of $(a+b)$ would be 2.

48. What is the HCF of the smallest composite number and the smallest prime number?

Ans :

The smallest prime number is 2 and the smallest composite number is $4 = 2^2$.
Hence, required HCF is $(2^2, 2) = 2$.

49. Calculate the HCF of $3^3 \times 5$ and $3^2 \times 5^2$.

Ans : [Board 2007]

We have
$$3^3 \times 5 = 3^2 \times 5 \times 3$$
$$3^2 \times 5^2 = 3^2 \times 5 \times 5$$
$$\text{HCF } (3^3 \times 5,\ 3^2 \times 5^2) = 3^2 \times 5$$
$$= 9 \times 5 = 45$$

50. If HCF $(a, b) = 12$ and $a \times b = 1,800$, then find LCM (a, b).

Ans :

We know that
$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$
Substituting the values we have
$$12 \times \text{LCM}(a, b) = 1800$$
or,
$$\text{LCM}(a,\ b) = \frac{1,800}{12} = 150$$

51. What is the condition for the decimal expansion of a rational number to terminate? Explain with the help of an example.

Ans :

The decimal expansion of a rational number terminates, if the denominator of rational number can be expressed as $2^m 5^n$ where m and n are non negative integers and p and q both co-primes.

e.g.
$$\frac{3}{10} = \frac{3}{2^1 \times 5^1} = 0.3$$

52. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal.

Ans : [Board Term-1 2016]

Since
$$\frac{1}{7} \times \frac{7}{100} = \frac{1}{100} = 0.01.$$
Thus smallest rational number is $\frac{7}{100}$

53. What type of decimal expansion does a rational number has? How can you distinguish it from decimal expansion of irrational numbers?

Ans : [Board Term-1 2016]

A rational number has its decimal expansion either terminating or non-terminating, repeating An irrational numbers has its decimal expansion non-repeating and non-terminating.

54. Calculate $\frac{3}{8}$ in the decimal form.

Ans : [Board 2008]

We have
$$\frac{3}{8} = \frac{3}{2^3} = \frac{2 \times 5^3}{2^3 \times 5^3}$$
$$= \frac{375}{10^3} = \frac{375}{1,000}$$
$$= 0.375$$

55. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

Ans : [Board 2009]

We have
$$\frac{6}{1250} = \frac{6}{2 \times 5^4} = \frac{6 \times 2^3}{2 \times 2^3 \times 5^4}$$
$$= \frac{6 \times 2^3}{2^4 \times 5^4} = \frac{6 \times 2^3}{(10)^4}$$
$$= \frac{48}{10000} = 0.0048$$
Thus $\frac{6}{1250}$ will terminate after 4 decimal places.

56. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

Ans : [Board 2010]

The required number is the LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
$$\text{LCM} = 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7$$
$$= 2520$$

57. Write whether rational number $\frac{7}{75}$ will have terminating decimal expansion or a non-terminating decimal.

Ans : [Board Term-1 2017, SQP]

We have
$$\frac{7}{75} = \frac{7}{3 \times 5^2}$$
Since denominator of given rational number is not of form $2^m \times 5^n$, Hence, It is non-terminating decimal expansion.

TWO MARKS QUESTIONS

58. If HCF of 144 and 180 is expressed in the form $13m-16$. Find the value of m.

Ans : [Board 2020 SQP Standard]

According to Euclid's algorithm any number a can be written in the form,

$$a = bq+r \text{ where } 0 \le r < b$$

Applying Euclid's division lemma on 144 and 180 we have

$$180 = 144 \times 1 + 36$$

$$144 = 36 \times 4 + 0$$

Here, remainder is 0 and divisor is 36. Thus HCF of 144 and 180 is 36.

Now $$36 = 13m - 16$$

$$36 + 16 = 13m$$

$$52 = 13m \Rightarrow m = 4$$

59. Find HCF and LCM of 404 and 96 and verify that HCF $\times$ LCM = Product of the two given numbers.

Ans : [Board 2018]

We have $$404 = 2 \times 2 \times 101$$

$$= 2^2 \times 101$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$= 2^5 \times 3$$

$$\text{HCF}(404,\ 96) = 2^2 = 4$$

$$\text{LCM}(404,\ 96) = 101 \times 2^5 \times 3 = 9696$$

$$\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$$

Also, $$404 \times 96 = 38784$$

Hence, HCF $\times$ LCM = Product of 404 and 96

60. Find HCF of the numbers given below:
$k, 2k, 3k, 4k$ and $5k$,, where k is a positive integer.

Ans : [Board Term-1 2015, Set-FHN8MGD]

Here we can see easily that k is common factor between all and this is highest factor Thus HCF of $k, 2k, 3k, 4k$ and $5k$, is k.

61. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

Ans : [Board Term-1 2012]

We have $$90 = 9 \times 10 = 9 \times 2 \times 5$$

$$= 2 \times 3^2 \times 5$$

and $$144 = 16 \times 9$$

$$= 2^4 \times 3^2$$

$$\text{HCF} = 2 \times 3^2 = 18$$

$$\text{LCM} = 2^4 \times 3^2 \times 5 = 720$$

62. Given that HCF $(306,\ 1314) = 18$. Find LCM $(306,\ 1314)$

Ans : [Board Term-1 2013]

We have $$\text{HCF}\ (306,\ 1314) = 18$$

$$\text{LCM}\ (306,\ 1314) = ?$$

Let $a = 306$ and $b = 1314$, then we have

$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$$

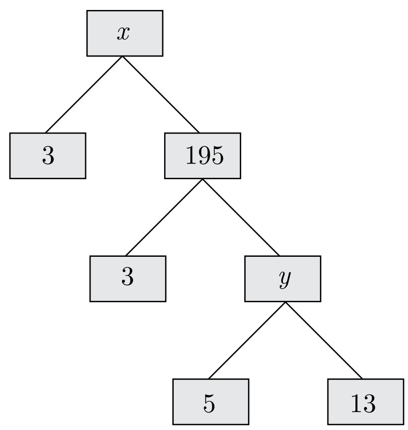
Substituting values we have

$$\text{LCM}(a, b) \times 18 = 306 \times 1314$$

$$\text{LCM}\ (a, b) = \frac{306 \times 1314}{18}$$

$$\text{LCM}\ (306, 1314) = 22{,}338$$

63. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1 2015]

We have $$y = 5 \times 13 = 65$$

and $$x = 3 \times 195 = 585$$

64. Explain why $(7 \times 13 \times 11) + 11$ and $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$ are composite numbers.

Ans : [Board Term-1 2012, Set-64]

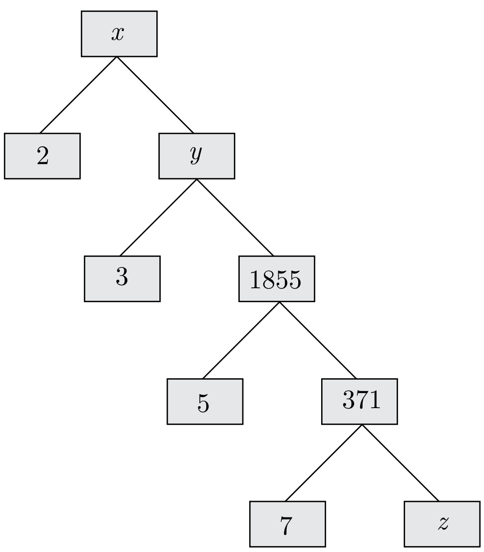
$$(7 \times 13 \times 11) + 11 = 11 \times (7 \times 13 + 1)$$
$$= 11 \times (91 + 1)$$
$$= 11 \times 92$$

and

$$(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$$
$$= 3(7 \times 6 \times 5 \times 4 \times 2 \times 1 + 1)$$
$$= 3 \times (1681) = 3 \times 41 \times 41$$

Since given numbers have more than two prime factors, both number are composite.

65. Complete the following factor tree and find the composite number x



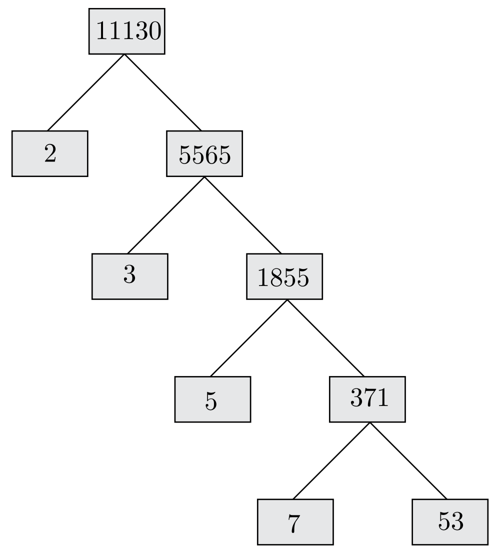
Ans : [Board Term-1 2015, Set DDE-M]

We have $z = \frac{371}{7} = 53$

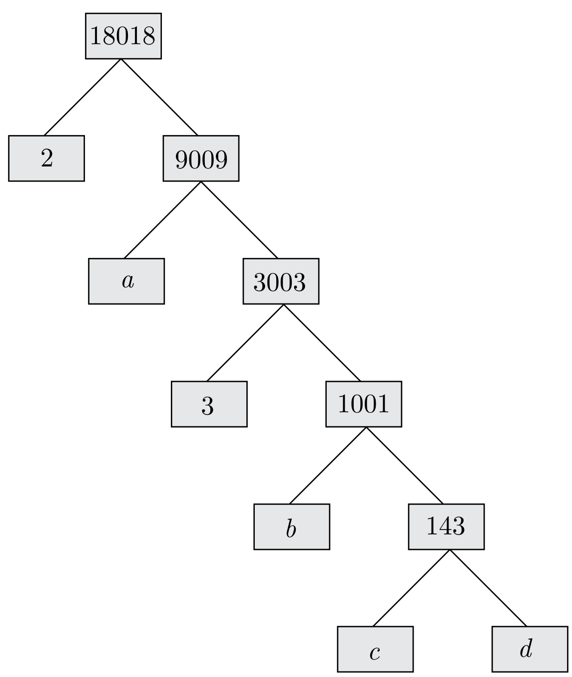
$$y = 1855 \times 3 = 5565$$

$$x = 2 \times y = 2 \times 5565 = 11130$$

Thus complete factor tree is as given below.



66. Find the missing numbers a, b, c and d in the given factor tree:



Ans : [Board Term-1 2012]

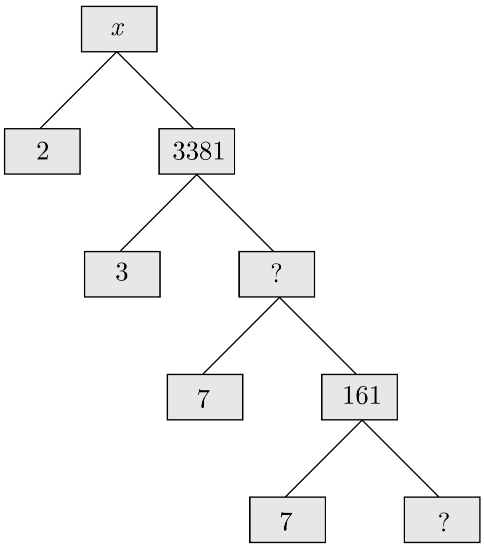
We have $a = \frac{9009}{3003} = 3$

$$b = \frac{1001}{143} = 7$$

Since $143 = 11 \times 13$,

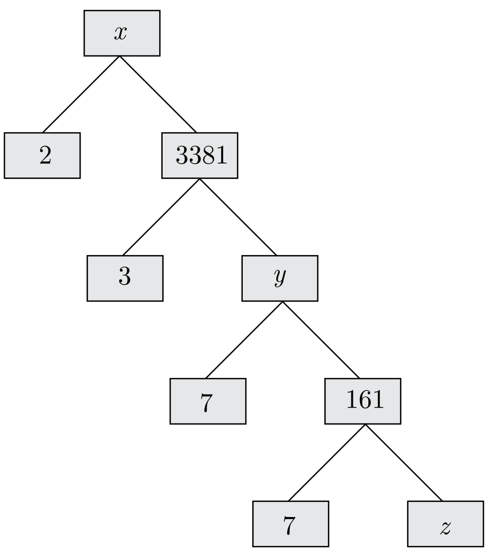
Thus $c = 11$ and $d = 13$ or $c = 13$ and $d = 11$

67. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1 2015, 2014]

We complete the given factor tree writing variable y and z as following.



We have $z = \frac{161}{7} = 23$

$y = 7 \times 161 = 1127$

Composite number, $x = 2 \times 3381 = 6762$

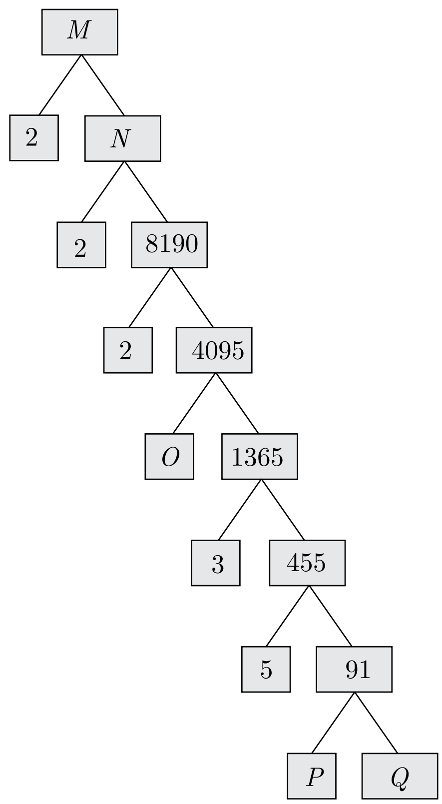
68. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

Ans : [Board Term-1 2016-17 Set; 193RQTQ, 2015, DDE-E]

A prime number (or a prime) is a natural number greater than 1 that cannot be formed by multiplying two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. For example, 5 is prime because the only ways of writing it as a product, 1×5 or 5×1, involve 5 itself. However, 6 is composite because it is the product of two numbers (2×3) that are both smaller than 6. Every composite number can be written as the product of two or more (not necessarily distinct) primes.

$$\begin{aligned}3 \times 12 \times 101 + 4 &= 4(3 \times 3 \times 101 + 1) \\ &= 4(909 + 1) \\ &= 4(910) \\ &= 2 \times 2 \times (10 \times 7 \times 13) \\ &= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \\ &= \text{a composite number}\end{aligned}$$

69. Complete the factor-tree and find the composite number M.



Ans : [Board Term-1 2013]

We have $91 = P \times Q = 7 \times 13$

So $P = 7, Q = 13$ or $P = 13, Q = 7$

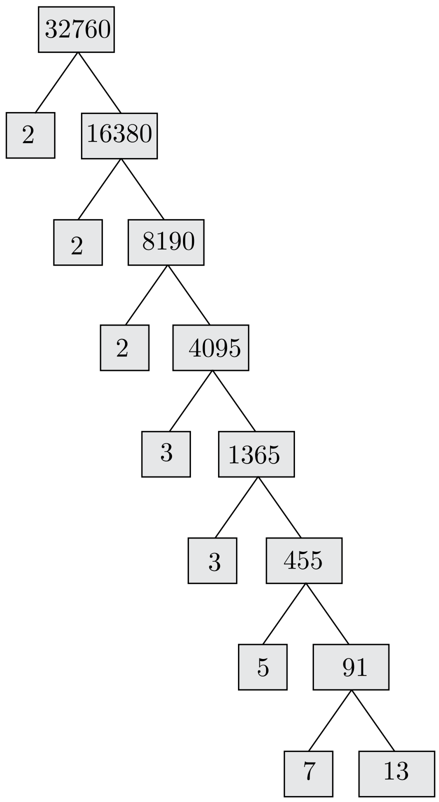
$$O = \frac{4095}{1365} = 3$$

$$N = 2 \times 8190 = 16380$$

Composite number,

$$M = 16380 \times 2 = 32760$$

Thus complete factor tree is shown below.



70. Find the smallest natural number by which 1200 should be multiplied so that the square root of the product is a rational number.

Ans : [Board Term-1 2016, 2015]

We have
$$1200 = 12 \times 100$$
$$= 4 \times 3 \times 4 \times 25$$
$$= 4^2 \times 3 \times 5^2$$

Here if we multiply by 3, then its square root will be $4 \times 3 \times 5$ which is a rational number. Thus the required smallest natural number is 3.

71. Can two numbers have 15 as their HCF and their LCM? Give reasons.

Ans :

LCM of two numbers should be exactly divisible by their HCF. Since, 15 does not divide 175, two numbers cannot have their HCF as 15 and LCM as 175.

72. Check whether 4^n can end with the digit 0 for any natural number n.

Ans : [Board Term-1 2015, Set-FHN8MGD; NCERT]

If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $4^n = 2^{2n}$ is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero. Hence 4^n cannot end with the digit zero.

73. Show that 7^n cannot end with the digit zero, for any natural number n.

Ans : [Board Term-1 2012, Set-63]

If the number 7^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 7^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $7^n = (1 \times 7)^n$ is 7. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 7^n. So, there is no natural number n for which 7^n ends with the digit zero. Hence

7^n cannot end with the digit zero.

74. Check whether $(15)^n$ can end with digit 0 for any $n \in N$.

Ans : [Board Term-1 2012]

If the number $(15)^n$, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of $(15)^n$ would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(15)^n = (3 \times 5)^n$ are 3 and 5. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(15)^n$. Since there is no prime factor 2, $(15)^n$ cannot end with the digit zero.

75. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

Ans : [Board Term-1 2016]

Here we have to determine the HCF of all length which can measure all dimension.

$$\text{Length, } \quad l = 8 \text{ m } 50 \text{ cm} = 850 \text{ cm}$$
$$= 50 \times 17 = 2 \times 5^2 \times 17$$
$$\text{Breadth, } \quad b = 6 \text{ m } 25 \text{ cm} = 625 \text{ cm}$$
$$= 25 \times 25 = 5^2 \times 5^2$$
$$\text{Height, } \quad h = 4 \text{ m } 75 \text{ cm} = 475 \text{ cm}$$
$$= 25 \times 19 = 5^2 \times 19$$
$$\text{HCF}(l, b, h) = \text{HCF }(850, 625, 475)$$
$$= \text{HCF }(2 \times 5^2 \times 17, \ \ 5^2, \ \ 5^2 \times 19)$$
$$= 5^2 = 25 \text{ cm}$$

Thus 25 cm rod can measure the dimensions of the room exactly. This is longest rod that can measure exactly.

76. Show that $5\sqrt{6}$ is an irrational number.

Ans : [Board Term-1 2015]

Let $5\sqrt{6}$ be a rational number, which can be expressed as $\frac{a}{b}$, where $b \neq 0$; a and b are co-primes.

$$\text{Now} \quad 5\sqrt{6} = \frac{a}{b}$$
$$\sqrt{6} = \frac{a}{5b}$$
$$\text{or,} \quad \sqrt{6} = \text{rational}$$

But, $\sqrt{6}$ is an irrational number. Thus, our assumption is wrong. Hence, $5\sqrt{6}$ is an irrational number.

77. Write the denominator of the rational number $\frac{257}{500}$ in the form $2^m \times 5^n$, where m and n are non-negative integers. Hence write its decimal expansion without actual division.

Ans :

$$\text{We have} \quad 500 = 25 \times 20$$
$$= 5^2 \times 5 \times 4$$
$$= 5^3 \times 2^2$$

Here denominator is 500 which can be written as $2^2 \times 5^3$.

Now decimal expansion,

$$\frac{257}{500} = \frac{257 \times 2}{2 \times 2^2 \times 5^3} = \frac{514}{10^3}$$
$$= 0.514$$

78. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Ans : [K.V.S.]

We have $\sqrt{2} = \sqrt{\frac{200}{100}}$ and $\sqrt{3} = \sqrt{\frac{300}{100}}$

We need to find a rational number x such that

$$\frac{1}{10}\sqrt{200} < x < \frac{1}{10}\sqrt{300}$$

Choosing any perfect square such as 225 or 256 in between 200 and 300, we have

$$x = \sqrt{\frac{225}{100}} = \frac{15}{10} = \frac{5}{3}$$

Similarly if we choose 256, then we have

$$x = \sqrt{\frac{256}{100}} = \frac{16}{10} = \frac{8}{5}$$

79. Write the rational number $\frac{7}{75}$ will have a terminating decimal expansion. or a non-terminating repeating decimal.

Ans : [Board 2018 SQP]

$$\text{We have} \quad \frac{7}{75} = \frac{7}{3 \times 5^2}$$

The denominator of rational number $\frac{7}{75}$ can not be written in form $2^m 5^n$ So it is non-terminating repeating decimal expansion.

80. Show that 571 is a prime number.

Ans :

$$\text{Let} \quad x = 571$$
$$\sqrt{x} = \sqrt{571}$$

Now 571 lies between the perfect squares of $(23)^2 = 529$ and $(24)^2 = 576$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Here 571 is not divisible by any of the above numbers, thus 571 is a prime number.

81. If two positive integers p and q are written as $p = a^2 b^3$ and $q = a^3 b$, where a and b are prime numbers than verify $\text{LCM}(p, q) \times \text{HCF}(q, q) = pq$

Ans : [Sample Paper 2017]

We have $\quad p = a^2 b^3 = a \times a \times b \times b \times b$

and $\quad q = a^3 b = a \times a \times a \times b$

Now $\quad \text{LCM}\ (p, q) = a \times a \times a \times b \times b \times b$

$$= a^3 b^3$$

and $\quad \text{HCF}\ (p, q) = a \times a \times b$

$$= a^2 b$$

$$\text{LCM}\ (p, q) \times \text{HCF}(p, q) = a^3 b^3 \times a^2 b$$

$$= a^5 b^4$$

$$= a^2 b^3 \times a^3 b$$

$$= pq$$

THREE MARKS QUESTIONS

82. An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Ans : [Board 2020 Delhi Basic]

Let the number of columns be x which is the largest number, which should divide both 612 and 48. It means x should be HCF of 612 and 48.

We can write 612 and 48 as follows

$$612 = 2 \times 2 \times 3 \times 3 \times 5 \times 17$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{HCF}(612,\ 28) = 2 \times 2 \times 3 = 12$$

Thus HCF of 104 and 96 is 12 i.e. 12 columns are required.

Here we have solved using Euclid's algorithm but you can solve this problem by simple mehtod of HCF.

83. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number.

Ans : [Board 2020 SQP Standard]

Assume that $2\sqrt{5} - 3$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $\quad 2\sqrt{5} - 3 = \dfrac{p}{q}$

where $q \neq 0$ and p and q are co-prime integers.

Rewriting the above expression as,

$$2\sqrt{5} = \frac{p}{q} + 3$$

$$\sqrt{5} = \frac{p + 3q}{2q}$$

Here $\frac{p+3q}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{5}$ should be a rational number. But $\sqrt{5}$ is irrational. This contradicts the given fact that $\sqrt{5}$ is irrational. Hence $2\sqrt{5} - 3$ is an irrational number.

84. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Ans : [Board 2019 Delhi]

Assume that $\frac{2+\sqrt{3}}{5}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$\frac{2+\sqrt{3}}{5} = \frac{p}{q}$$

$$2 + \sqrt{3} = \frac{5p}{q}$$

$$\sqrt{3} = \frac{5p}{q} - 2$$

$$\sqrt{3} = \frac{5p - 2q}{q}$$

Since, p and q are co-prime integers, then $\frac{5p-2q}{q}$ is a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. So, our assumption is wrong. Therefore $\frac{2+\sqrt{3}}{5}$ is an irrational number.

85. Given that $\sqrt{3}$ is irrational, prove that $(5 + 2\sqrt{3})$ is an irrational number.

Ans : [Board 2020 Delhi Basic]

Assume that $(5 + 2\sqrt{3})$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p

and q are co-prime integers and $q \neq 0$.

Now $$5+2\sqrt{3} = \frac{p}{q}$$

where $q \neq 0$ and p and q are integers.

Rewriting the above expression as,

$$2\sqrt{3} = \frac{p}{q} - 5$$

$$\sqrt{3} = \frac{p-5q}{2q}$$

Here $\frac{p-5q}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{3}$ should be a rational number. But $\sqrt{3}$ is irrational. This contradicts the given fact that $\sqrt{3}$ is irrational. Hence $(5+2\sqrt{3})$ is an irrational number.

86. Prove that $2+5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Ans : [Board 2019 OD]

Assume that $2+5\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$2+5\sqrt{3} = \frac{p}{q}, \quad q \neq 0$$

$$5\sqrt{3} = \frac{p}{q} - 2$$

$$5\sqrt{3} = \frac{p-2q}{q}$$

$$\sqrt{3} = \frac{p-2q}{5q}$$

Here $\sqrt{3}$ is irrational and $\frac{p-2q}{5q}$ is rational because p and q are co-prime integers. But rational number cannot be equal to an irrational number. Hence $2+5\sqrt{3}$ is an irrational number.

87. Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number.

Ans : [Board 2018]

Assume that $(5+3\sqrt{2})$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $$5+3\sqrt{2} = \frac{p}{q}$$

where $q \neq 0$ and p and q are integers.

Rewriting the above expression as,

$$3\sqrt{2} = \frac{p}{q} - 5$$

$$\sqrt{2} = \frac{p-5q}{3q}$$

Here $\frac{p-5q}{3q}$ is rational because p and q are co-prime integers, thus $\sqrt{2}$ should be a rational number. But $\sqrt{2}$ is irrational. This contradicts the given fact that $\sqrt{2}$ is irrational. Hence $(5+3\sqrt{2})$ is an irrational number.

88. Write the smallest number which is divisible by both 306 and 657.

Ans : [Board 2019 OD]

The smallest number that is divisible by two numbers is obtained by finding the LCM of these numbers Here, the given numbers are 306 and 657.

$$306 = 6 \times 51 = 3 \times 2 \times 3 \times 17$$

$$657 = 9 \times 73 = 3 \times 3 \times 73$$

$$\text{LCM}(306, 657) = 2 \times 3 \times 3 \times 17 \times 73$$

$$= 22338$$

Hence, the smallest number which is divisible by 306 and 657 is 22338.

89. Show that numbers 8^n can never end with digit 0 of any natural number n.

Ans :

If the number 8^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2. That is, the prime factorization of 8^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(8)^n = (2^3)^n = 2^{3n}$ is 2. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(8)^n$. Since there is no prime factor 5, $(8)^n$ cannot end with the digit zero.

90. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Ans : [Board Term-1 2011]

The required answer will be HCF of 144 and 90.

$$144 = 2^4 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{HCF}(144, 90) = 2 \times 3^2 = 18$$

Thus each stack would have 18 cartons.

91. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Ans : [Board Term-1 2011, Set-44]

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$9 = 3 \times 3 = 3^2$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5$$

$$= 150 \text{ minutes}$$

The bells will toll next together after 180 minutes.

92. Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

Ans :

Finding prime factor of given number we have,

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

$$\text{HCF}(16, 36) = 2 \times 2 = 4$$

$$\text{LCM}(16, 36) = 2^4 \times 3^2$$

$$= 16 \times 9 = 144$$

Check :

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

or, $$4 \times 144 = 16 \times 36$$

$$576 = 576$$

Thus $$\text{LHS} = \text{RHS}$$

93. Find the HCF and LCM of 510 and 92 and verify that HCF $\times$ LCM $=$ Product of two given numbers.

Ans : [Board Term-1 2011]

Finding prime factor of given number we have,

$$92 = 2^2 \times 23$$

$$510 = 30 \times 17 = 2 \times 3 \times 5 \times 17$$

$$\text{HCF}(510, 92) = 2$$

$$\text{LCM}(510, 92 = 2^2 \times 23 \times 3 \times 5 \times 14$$

$$= 23460$$

$$\text{HCF}(510, 92) \times \text{LCM}(510, 92)$$

$$= 2 \times 23460 = 46920$$

Product of two numbers $= 510 \times 92 \quad 46920$

Hence, $\text{HCF} \times \text{LCM} =$ Product of two numbers

94. The HCF of 65 and 117 is expressible in the form $65m–117$. Find the value of m. Also find the LCM of 65 and 117 using prime factorization method.

Ans : [Board Term-1 2011, Set-40]

Finding prime factor of given number we have.

$$117 = 13 \times 2 \times 3$$

$$65 = 13 \times 5$$

$$\text{HCF}(117, 65) = 13$$

$$\text{LCM}(117, 65) = 13 \times 5 \times 3 \times 3 = 585$$

$$\text{HCF} = 65m - 117$$

$$13 = 65m–117$$

$$65m = 117 + 13 = 130$$

$$m = \frac{130}{65} = 2$$

95. Express $\left(\frac{15}{4} + \frac{5}{40}\right)$ as a decimal fraction without actual division.

Ans : [Board Term-1 2011, Set-74]

We have $$\frac{15}{4} + \frac{5}{40} = \frac{15}{4} \times \frac{25}{25} + \frac{5}{40} \times \frac{25}{25}$$

$$= \frac{375}{100} + \frac{125}{1000}$$

$$= 3.75 + 0.125 = 3.875$$

96. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

Ans : [Board Term-1 2011, Set-A1]

Let $$x = 0.3\overline{178}$$

$$x = 0.3178178178$$

$$10,000x = 3178.178178...$$

$$10x = 3.178178....$$

Subtracting, $$9990x = 3175$$

or, $$x = \frac{3175}{9990} = \frac{635}{1998}$$

97. Prove that $\sqrt{2}$ is an irrational number.

Ans : [Board Term-1 2011, NCERT]

Let $\sqrt{2}$ be a rational number.

Then $$\sqrt{2} = \frac{p}{q},$$

where p and q are co-prime integers and $q \neq 0$ On squaring both the sides we have,

$$2 = \frac{p^2}{q^2}$$

or, $$p^2 = 2p^2$$

Since p^2 is divisible by 2, thus p is also divisible by 2.

Let $p = 2r$ for some positive integer r, then we have

$$p^2 = 4r^2$$

$$2q^2 = 4r^2$$

or, $$q^2 = 2r^2$$

Since q^2 is divisible by 2, thus q is also divisible by 2.

We have seen that p and q are divisible by 2, which contradicts the fact that p and q are co-primes. Hence, our assumption is false and $\sqrt{2}$ is irrational.

98. If p is prime number, then prove that $\sqrt{p}$ is an irrational.

Ans : [Board Term-1 2013]

Let p be a prime number and if possible, let $\sqrt{p}$ be rational

Thus $$\sqrt{p} = \frac{m}{n},$$

where m and n are co-primes and $n \neq 0$.
Squaring on both sides, we get

$$p = \frac{m^2}{n^2}$$

or, $$pn^2 = m^2 \quad ...(1)$$

Here p divides pn^2. Thus p divides m^2 and in result p also divides m.

Let $m = pq$ for some integer q and putting $m = pq$ in eq. (1), we have

$$pn^2 = p^2q^2$$

or, $$n^2 = pq^2$$

Here p divides pq^2.Thus p divides n^2 and in result p also divides n.

[$\because$ p is prime and p divides $n^2 \Rightarrow p$ divides n]

Thus p is a common factor of m and n but this contradicts the fact that m and n are primes. The contradiction arises by assuming that $\sqrt{p}$ is rational.

Hence, $\sqrt{p}$ is irrational.

99. Prove that $3+\sqrt{5}$ is an irrational number.

Ans :

Assume that $3+\sqrt{5}$ is a rational number, then we have

$$3+\sqrt{5} = \frac{p}{q}, \quad q \neq 0$$

$$\sqrt{5} = \frac{p}{q} - 3$$

$$\sqrt{5} = \frac{p-3q}{q}$$

Here $\sqrt{5}$ is irrational and $\frac{p-3q}{q}$ is rational. But rational number cannot be equal to an irrational number. Hence $3+\sqrt{5}$ is an irrational number.

100. Prove that $\sqrt{5}$ is an irrational number and hence show that $2-\sqrt{5}$ is also an irrational number.

Ans : [Board Term-1 2011]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b}, \qquad (a, b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

Let us assume that $2-\sqrt{5}$ be rational equal to a, then we have

$$2-\sqrt{5} = a$$

$$2-a = \sqrt{5}$$

Since we have assume $2-a$ is rational, but $\sqrt{5}$ is not rational. Rational number cannot be equal to an irrational number. Thus $2-\sqrt{5}$ is irrational.

101. Show that exactly one of the number $n, n+2$ or $n+4$ is divisible by 3.

Ans : [Sample Paper 2017]

If n is divisible by 3, clearly $n+2$ and $n+4$ is not divisible by 3.

If n is not divisible by 3, then two case arise as given below.

Case 1: $n = 3k+1$

$$n+2 = 3k+1+2 = 3k+3 = 3(k+1)$$

and $$n+4 = 3k+1+4 = 3k+5 = 3(k+1)+2$$

We can clearly see that in this case $n+2$ is divisible by 3 and $n+4$ is not divisible by 3. Thus in this case only $n+2$ is divisible by 3.

Case 1: $n = 3k+2$

$$n+2 = 3k+2+2 = 3k+4 = 3(k+1)+1$$

and $$n+4 = 3k+2+4 = 3k+6 = 3(k+2)$$

We can clearly see that in this case $n+4$ is divisible by 3 and $n+2$ is not divisible by 3. Thus in this case only $n+4$ is divisible by 3.

Hence, exactly one of the numbers $n, n+2, n+4$ is divisible by 3.

FOUR MARKS QUESTIONS

102. Prove that $\sqrt{3}$ is an irrational number.

Ans : [Board 2020 OD Basic]

Assume that $\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{a}{b}$ where a and b are co-prime integers and $q \neq 0$.

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

Now $$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 3b^2$ we have

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

103. Prove that $\sqrt{5}$ is an irrational number.

Ans : [Board 2020 OD Standard]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

104. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF $\times$ LCM of these numbers equal to the product of the given three numbers?

Ans :

Finding prime factor of given number we have,

$$378 = 2 \times 3^3 \times 7$$

$$180 = 2^2 \times 3^2 \times 5$$

$$420 = 2^2 \times 3 \times 7 \times 5$$

$$\text{HCF}(378,\ 180,\ 420) = 2 \times 3 = 6$$

$$\text{LCM}(378, 180, 420) = 2^2 \times 3^3 \times 5 \times 7$$
$$= 2^2 \times 3^3 \times 5 \times 7 = 3780$$
$$\text{HCF} \times \text{LCM} = 6 \times 3780 = 22680$$

Product of given numbers

$$= 378 \times 180 \times 420$$
$$= 28576800$$

Hence, HCF $\times$ LCM $\neq$ Product of three numbers.

105. State Fundamental theorem of Arithmetic. Find LCM of numbers 2520 and 10530 by prime factorization by 3.

Ans : [Board Term-1 2016]

The fundamental theorem of arithmetic (FTA), also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is prime itself or is the product of a unique combination of prime numbers.

OR

Every composite number can be expressed as the product powers of primes and this factorization is unique.

Finding prime factor of given number we have,

$$2520 = 20 \times 126 = 20 \times 6 \times 21$$
$$= 2^3 \times 3^2 \times 5 \times 7$$
$$10530 = 30 \times 351 = 30 \times 9 \times 39$$
$$= 30 \times 9 \times 3 \times 13$$
$$= 2 \times 3^4 \times 5 \times 13$$
$$\text{LCM}(2520, 10530) = 2^3 \times 3^4 \times 5 \times 7 \times 13$$
$$= 294840$$

106. Can the number 6^n, n being a natural number, end with the digit 5 ? Give reasons.

Ans : [Board Term-1 2015]

If the number 6^n for any n, were to end with the digit five, then it would be divisible by 5. That is, the prime factorization of 6^n would contain the prime 5. This is not possible because the only prime in the factorization of $6^n = (2 \times 3)^n$ are 2 and 3. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 6^n. Since there is no prime factor 5, 6^n cannot end with the digit five.

107. State Fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

Ans : [Board Term-1 2015]

Fundamental theorem of Arithmetic : Every integer greater than one ither is prime itself or is the product of prime numbers and that this product is unique. Up to the order of the factors.

LCM of two numbers should be exactly divisible by their HCF. In other words LCM is always a multiple of HCF. Since, 24 does not divide 540 two numbers cannot have their HCF as 24 and LCM as 540.

$$\text{HCF} = 24$$
$$\text{LCM} = 540$$
$$\frac{LCM}{HCF} = \frac{540}{24} = 22.5 \text{ not an integer}$$

108. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Ans :

We have $n^3 - n = n(n^2 - 1)$

$$= (n-1)n(n+1)$$
$$= (n-1)n(n+1)$$

Thus $n^3 - n$ is product of three consecutive positive integers.

Since, any positive integers a is of the form $3q, 3q+1$ or $3q+2$ for some integer q.

Let $a, a+1, a+2$ be any three consecutive integers.

Case I : $a = 3q$

If $a = 3q$ then,

$$a(a+1)(a+2) = 3q(3q+1)(3q+2)$$

Product of two consecutive integers $(3q+1)$ and $(3q+2)$ is an even integer, say $2r$.

Thus $a(a+1)(a+2) = 3q(2r)$

$$= 6qr, \text{ which is divisible by 6.}$$

Case II : $a = 3q+1$

If $a = 3q+1$ then

$$a(a+1)(a+2) = (3q+1)(3q+2)(3q+3)$$
$$= (2r)(3)(q+1)$$
$$= 6r(q+1)$$

which is divisible by 6.

Case III : $a = 3q+2$

If $a = 3q+2$ then

$$a(a+1)(a+2) = (3q+2)(3q+3)(3q+4)$$
$$= 3(3q+2)(q+1)(3q+4)$$

Here $(3q+2)$ and $= 3(3q+2)(q+1)(3q+4)$

$= $ multiple of 6 every q

$= 6r$ (say)

which is divisible by 6. Hence, the product of three consecutive integers is divisible by 6 and $n^3 - n$ is also divisible by 3.

109. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

Ans : [Board Term-1 2012 Set-25]

We have $n^2 - n = n(n-1)$

Thus $n^2 - n$ is product of two consecutive positive integers.

Any positive integer is of the form $2q$ or $2q+1$, for some integer q.

Case 1 : $n = 2q$

If $n = 2q$ we have

$$n(n-1) = 2q(2q-1)$$
$$= 2m,$$

where $m = q(2q-1)$ which is divisible by 2.

Case 1 : $n = 2q+1$

If $n = 2q+1$, we have

$$n(n-1) = (2q+1)(2q+1-1)$$
$$= 2q(2q+1)$$
$$= 2m$$

where $m = q(2q+1)$ which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

110. Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7+2\sqrt{3}$ is also an irrational number.

Ans : [Board Term-1 2012]

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b}, \qquad (a, b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 9b^2$ we have

$$3b^2 = 9c^2$$
$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

Let us assume that $7+2\sqrt{3}$ be rational equal to a, then we have

$$7+2\sqrt{3} = \frac{p}{q} \qquad q \neq 0 \text{ and } p \text{ and } q \text{ are co-primes}$$

$$2\sqrt{3} = \frac{p}{q} - 7 = \frac{p-7q}{q}$$

or $$\sqrt{3} = \frac{p-7q}{2q}$$

Here $p-7q$ and $2q$ both are integers, hence $\sqrt{3}$ should be a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. Hence our assumption is not correct and $7+2\sqrt{3}$ is irrational.

111. Show that there is no positive integer n, for which $\sqrt{n-1}+\sqrt{n-1}$ is rational.

Ans : [Board Term-1 2012]

Let us assume that there is a positive integer n for which $\sqrt{n-1}+\sqrt{n-1}$ is rational and equal to $\frac{p}{q}$, where p and q are positive integers and $(q \neq 0)$.

$$\sqrt{n-1}+\sqrt{n-1} = \frac{p}{q} \qquad ...(1)$$

or, $$\frac{q}{p} = \frac{1}{\sqrt{n-1}+\sqrt{n+1}}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(\sqrt{n-1}+\sqrt{n+1})(\sqrt{n-1}-\sqrt{n+1})}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)}$$

or $$\frac{q}{p} = \frac{\sqrt{n-1}-\sqrt{n+1}}{-2}$$

$$\sqrt{n+1}-\sqrt{n-1} = \frac{2q}{p} \qquad ...(2)$$

Adding (1) and (2), we get

$$2\sqrt{n+1} = \frac{p}{q}+\frac{2q}{p} = \frac{p^2+2q^2}{pq} \qquad ...(3)$$

Subtracting (2) from (1) we have

$$2\sqrt{n-1} = \frac{p^2-2q^2}{pq} \qquad ...(4)$$

From (3) and (4), we observe that $\sqrt{n+1}$ and $\sqrt{n-1}$ both are rational because p and q both are rational. But it possible only when $(n+1)$ and $(n-1)$ both are perfect squares. But they differ by 2 and two perfect squares never differ by 2. So both $(n+1)$ and $(n-1)$ cannot be perfect squares, hence there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.